EJONS 8(4) Pages: 390-394

EJONS

International Journal on Mathematic, Engineering and Natural Sciences

(Uluslararası Fen,Mühendislik ve Doğa Bilimleri Dergisi)

https://ejons.org/index.php/ejons

e-ISSN: 2602 - 4136

Research Article

Doi: https://doi.org/10.5281/zenodo.14039850

Genişletilmiş Zamansal Ayarda Kutupsuz Gluon İlerleticileri

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Makale Tarihçesi

Geliş: 28.09.2024
Kabul: 08.11.2024

Anahtar Kelimeler

Ayar Kuantizasyonu,
Zamansal Ayar,
Gluon Green Fonksiyonu

Öz: Yang-Mills teorilerinin formülasyonunda temel ilke, Lagranjiyenin yerel ayar dönüşümleri altında değişmezliğidir. Ancak, bu ayar simetrisi, ayar alanlarının kuantizasyonunda, özellikle yol integral formülasyonunda fiziksel olmayan serbestlik derecelerinin dahil edilmesi nedeniyle önemli zorluklar yaratır. Faddeev-Popov yöntemi, bu fiziksel olmayan modları ortadan kaldıran bir ayar koşulu uygulayarak bu zorlukları ele almak için yaygın olarak kullanılmaktadır. Bu çalışma, özellikle zamansal ayar olmak üzere, Yang-Mills alanlarının kovaryant olmayan ayarlarda kuantizasyonuna odaklanmaktadır. Kovaryant olmayan ayarlar, döngü hesaplamaları sırasında hayalet alanları ayrıştırmaları nedeniyle avantaj sağlarlar. Bununla birlikte, özellikle gluon Green fonksiyonlarında $\mathbf{1}/(\boldsymbol{n}\cdot\boldsymbol{k})$ biçiminde kutupların veya tekilliklerin ortaya çıkması gibi kendine özgü zorlukları da beraberinde getirirler. Bu kutupların ele alınması basit değildir ve literatürde çeşitli yöntemler önerilmiştir. Bu çalışmada, Veliev, Karnaukhov ve Fainberg tarafından geliştirilen ve bu kutupları etkili bir şekilde ortadan kaldırmak için ayar koşulunu genişleten tekniği kullanmaktayız; bu sayede düzenlenmiş ve kutupsuz bir gluon propagatörü elde edilmektedir. Araştırmamız, $(\boldsymbol{n}_\mu+\boldsymbol{\varepsilon}^2\boldsymbol{k}_\mu/(\boldsymbol{n}\cdot\boldsymbol{k}))\boldsymbol{A}_\mu^a=\mathbf{0}$ zamansal ayara genişletilmiş bir versiyon kullanarak bu tekilliklerden arındırılmış Green fonksiyonları türettiğimiz bir yaklaşımı incelemektedir.

Atıf Künyesi: Alemdar, Ö.F., Yılmazkaya Süngü, J., ve Veliev, E.V. (2024). Genişletilmiş Zamansal Ayarda Kutupsuz Gluon İlerleticileri, EJONS International Journal on Mathematic, Engineering and Natural Sciences 8(4), 390-394. **How To Cite:** Alemdar, Ö.F., Yılmazkaya Süngü, J., & Veliev, E.V. (2024 Pole-Free Gluon Propagators In Extended Temporal Gauge, EJONS International Journal on Mathematic, Engineering and Natural Sciences 8(4), pages: 390-394.

Pole-Free Gluon Propagators In Extended Temporal Gauge

Article Info

Received: 28.09.2024
Accepted: 08.11.2024

Abstract: A central principle in the formulation of Yang-Mills theories is the invariance of the Lagrangian under local gauge transformations. However, this gauge symmetry introduces significant challenges in the quantization of gauge fields, primarily due to the inclusion of non-physical degrees of freedom in the path integral formalism. The Faddeev-Popov method is widely used to address these challenges by imposing a gauge condition, which eliminates these non-physical modes. This work focuses on the quantization of Yang-Mills fields in noncovariant gauges, specifically the temporal gauge. Noncovariant gauges are advantageous in that they decouple ghost fields during loop calculations. Nevertheless, they come with their own set of complications, notably the appearance of singularities or poles in gluon Green functions, particularly of the form $1/(n \cdot k)$. The treatment of these poles is not straightforward and has been a topic of various proposed methods in the literature. In this study, we adopt the technique developed by Veliev, Karnaukhov, and Fainberg, which extends the gauge condition to effectively remove these poles, resulting in a regularized, pole-free gluon propagator. Our investigation explores an extended version of the temporal gauge $(n_\mu + \varepsilon^2 k_\mu/(n \cdot k))A^a_\mu = 0$,which smoothly transitions into the standard temporal gauge. Through this extension and after completing the standard quantization process, we derive Green functions that are free of singularities.

Keywords

Field Quantization,
Temporal Gauge,
Gluon Green Function

1.Introduction

The study of Yang-Mills theories stands as a cornerstone in modern theoretical physics, offering profound insights into the fundamental forces that govern the universe (Yang and Mills 1954). At the heart of these theories lies the principle of local gauge invariance, which underpins the construction of Lagrangian densities describing the interactions between gauge fields. However, the quantization of gauge fields within this framework presents formidable challenges, primarily due to the presence of non-physical degrees of freedom arising from gauge symmetries. One pivotal approach to overcome these challenges is the Faddeev-Popov method, which facilitates the elimination of non-physical degrees of freedom through the imposition of suitable gauge conditions (Ryder 1985, Peskin et al. 1995).

An effective Lagrangian is described as:

$$L_{eff} = L + \bar{c}^{a} \frac{\delta F^a}{\delta A^c_\mu} D^{cb}_\mu c^b + \frac{1}{2\alpha} F^a f F^a \,, \tag{1}$$

here, the L is the classical Lagrangian, the second term expresses the contribution of ghost fields, the third term in Eq. (1) denotes the contribution coming from gauge condition $F^a = 0$, A^c_μ are gluon fields and, f is a weight function. In this work, we chose a weight function in the following form in momentum space:

$$f = \frac{\alpha (n \cdot k)^2}{\varepsilon^4}, \tag{2}$$

where $n_\mu = (n_0, \vec{n})$ is an arbitrary constant four-vector, k shows the four-momentum and in Eq. (1) covariant derivative D^{cb}_μ is expressed as:

$$D^{cb}_\mu = \delta^{cb} \partial_\mu + g f^{acb} A^a_\mu. \tag{3}$$

The Green's function describes the response of a field to a source term $\mathcal{J}^a_\mu(x)$:

$$D^{ab}_{\mu\nu}(x - y) = \frac{\delta^2 (\ln Z)}{\delta \mathcal{J}^a_\mu(x) \delta \mathcal{J}^b_\nu(y)}. \tag{4}$$

$D^{ab}_{\mu\nu}(x - y)$ represents the Green function of the gluon field. $Z[\mathcal{J}]$ is the generating functional, δ denotes the functional derivative concerning the sources $\mathcal{J}^a_\mu(x)$ and $\mathcal{J}^b_\nu(y)$.

In this context, noncovariant gauges emerge as a crucial tool, offering both advantages and complexities in the quantization process. While these gauges simplify certain calculations, they also introduce intricacies, particularly regarding the treatment of poles in gluon Green functions. Addressing these issues requires innovative methodologies, such as the one proposed by Veliev, Karnaukhov, and Fainberg (Fainberg et al. 1989), which aims to render gluon propagators pole-free by extending the gauge condition. In this study, we embark on an exploration of Yang-Mills field quantization in noncovariant gauges, with a specific focus on the extended temporal gauge. Through a rigorous investigation, we seek to elucidate the quantization procedure, unraveling the complexities inherent in noncovariant gauges and laying the groundwork for a deeper understanding of gauge theories and their implications in theoretical physics (Leibbrandt 1994, Veliev 2001, Veliev and Yılmazkaya 2004, Veliev et al. 2018, Yılmazkaya 2004, Süngü et al. 2020, Süngü et al. 2022, Mutlu et al. 2022).

In this paper, we used a modified temporal gauge condition to get rid of the poles which leads to singularities in loop calculations. In the next section, we give some technical information to treat the poles. In section III, we present our results.

2. The Method

Poles in Quantum Chromodynamics (QCD) arise when the denominator of a Feynman diagram integrand goes to zero. These poles can cause infinities in calculations, which need to be addressed using regularization techniques. Two common techniques in noncovariant gauges are:

- **Mandelstam Leibbrandt (ML) prescription**: This prescription introduces a small, artificial term for the gluon propagator. This term acts as a regulator, pushing the pole away from zero and making the integral finite. After performing the calculation, this term is then set back to zero. The advantage of ML is its simplicity, but it can introduce difficulties in loop diagram calculations (Mandelstam 1983, Leibbrandt 1984). The ML prescription is:

$$\frac{1}{(n \cdot k)} = \lim_{\varepsilon \to 0} \frac{(k \cdot \bar{n})}{(n \cdot k)(k \cdot \bar{n}) + i\varepsilon}, \qquad \varepsilon > 0, \tag{5}$$

where $\bar{n}_{\mu}$ is a new null vector with $\bar{n}_{\mu} = (n_0, -\vec{n})$.

- **Principal Value (PV) Prescription**: This technique involves taking the average of the integrand's limit as the momentum approaches the pole from either side (positive and negative infinity). This approach cancels out the infinities but can sometimes lead to ambiguities, especially when multiple poles are involved. The standard PV regularization is applied at the level of the Feynman rules to the axial part of the gluon propagator:

$$\frac{1}{(n \cdot k)^{\ell}} \to \frac{1}{2} \left[\frac{1}{(n \cdot k + i\varepsilon)^{\ell}} + \frac{1}{(n \cdot k - i\varepsilon)^{\ell}}\right], \tag{6}$$

where k is four-momentum and ε is an infinitesimal regulator ('t Hooft 1979).

Both ML and PV prescriptions can be derived from the quantization procedure (Fainberg et al. 1989, Veliev 2001) and these receipts are tools to handle infinities arising from poles in QCD calculations. Each has its advantages and disadvantages:

ML prescription is conceptually simple but some complexities arise in loop diagram computations. PV prescription avoids unphysical effects but can be ambiguous in some cases. Additionally, dimensional regularization and momentum subtraction regularization techniques should be integrated with these mentioned prescriptions. These techniques can offer advantages over ML and PV prescriptions, such as preserving gauge invariance and allowing for easier calculations in some cases (Veliev 2001).

In Yang-Mills theories, the choice of gauge condition plays a crucial role in simplifying calculations and understanding the behavior of particles and fields as well. However, certain gauge condition choices, such as temporal gauges, can introduce singularities that complicate the analysis. Singularities arise due to residual symmetries present in the Lagrangian density after quantization, leading to unphysical results in calculations. To address these singularities and obtain meaningful physical predictions, various modification schemes have been proposed for temporal gauges.

One of these methods involves extending the temporal gauge condition to incorporate additional terms that eliminate the problematic singularities. By carefully adjusting the gauge condition, it is possible to derive modified propagators that are free from singularities in the desired limits. These modifications ensure that calculations in temporal gauges yield physically meaningful results, allowing for a deeper understanding of QCD phenomena, particularly in the infrared regime where singularities are most prevalent. The extended temporal gauge condition is:

$$\left(n^{\mu}+\varepsilon^{2}\frac{k^{\mu}}{n\cdot k}\right)A_{\mu}^{a}(x)=0\,;\;\; n^{2}=1, n_{\mu}=(1,0,0,0). \tag{7}$$

Taking into account condition (7) in Eq. (1) and after the standard quantization procedure we obtain the inverse gluon Green function in momentum space as follows:

$$D_{\mu v}^{-1}(k)=k^{2}g_{\mu v}+\frac{(n\cdot k)^{2}}{\varepsilon^{4}}\,n_{\mu}n_{v}+\frac{(n\cdot k)}{\varepsilon^{2}}\left(k_{\mu}n_{v}+k_{v}n_{\mu}\right). \tag{8}$$

In the above expression $D_{\mu v}^{-1}(k)$'s tensor structure leads us to the following general Lorentz structure:

$$D_{\mu v}(k)=F(k)g_{\mu v}+G(k)\left(n_{\mu}k_{v}+n_{v}k_{\mu}\right)+\;H(k)k_{\mu}k_{v}+U(k)n_{\mu}n_{v}. \tag{9}$$

The propagators $D_{\mu v}^{-1}(k)$ and $D_{\mu v}(k)$ represent the inverse and regular gluon propagators, respectively, in a noncovariant gauge. The product of the inverse and regular propagators yields the identity matrix:

$$D^{-1}(k)D(k)=I.$$

Additionally, the scalar functions $F(k), G(k), H(k)$ and $U(k)$ in Eq. (8) are obtained as:

$$F(k)=\frac{1}{k^{2}}\;;\quad G(k)=-\frac{1}{k^{2}}\frac{(n\cdot k)}{(n\cdot k)^{2}+\varepsilon^{2}k^{2}};$$

$$H(k)=\frac{1}{k^{2}}\frac{(n\cdot k)^{2}}{((n\cdot k)^{2}+\varepsilon^{2}k^{2})^{2}}\;\;;\quad U(k)=0.$$

Finally, we obtain the gluon propagator with:

$$D_{\mu v}(k)=\frac{1}{k^{2}}\left[g_{\mu v}-\frac{n_{\mu}k_{v}+n_{v}k_{\mu}}{(n\cdot k)^{2}+\varepsilon^{2}k^{2}}(n\cdot k)+\frac{(n\cdot k)^{2}}{((n\cdot k)^{2}+\varepsilon^{2}k^{2})^{2}}\;k_{\mu}k_{v}\right]. \tag{10}$$

In the limit as $\varepsilon\rightarrow 0$, the gluon propagator $D_{\mu v}(k)$ reduces to:

$$D_{\mu v}(k)=\frac{1}{k^{2}}\left[g_{\mu v}-\frac{n_{\mu}k_{v}+n_{v}k_{\mu}}{(n\cdot k)}+\frac{k_{\mu}k_{v}}{(n\cdot k)^{2}}\right] \tag{11}$$

4.Results

In conclusion, our study delves into the intricacies of Yang-Mills field quantization, particularly focusing on the challenges posed by noncovariant gauges. While the existence of gauge symmetries is fundamental to Yang-Mills theories, it introduces complexities during quantization due to the presence of non-physical degrees of freedom. We employ the Faddeev-Popov method to address these difficulties by eliminating these non-physical degrees of freedom through appropriate gauge conditions. Our investigation highlights the advantages and disadvantages of noncovariant gauges, such as simplifying loop calculations while complicating others. Moreover, we tackle the issue of gluon Green functions exhibiting poles in these gauges, proposing a method inspired by Veliev, Karnaukhov, and Fainberg (Fainberg et al. 1989) to eliminate these poles and obtain pole-free propagators. Specifically, we extend the gauge condition to achieve pole-free Green functions within an extended temporal gauge, which smoothly transitions to the standard temporal-gauge condition in the limit. Through rigorous analysis and application of this method, we successfully navigate the challenges of quantization in noncovariant gauges, paving the way for a deeper understanding of Yang-Mills theories and their applications in theoretical physics.

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